Assignment 1

HiOA ELTS2300 Dynamic systems

Due date: 03-10-2016 8:00AM.

# Heated chemical reactor

Consider a simplified model of a heated chemical reactor as in the figure:

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|  |  |  |
| --- | --- | --- |
| Symbol | Value / Units | Description |
|  | 4200 | Specific heat of water |
|  | 1000 | Water density |
|  | 0.2 | Water flow (in and out) in liters per second |
|  | 1 | Tank volume |
|  | 500 | Heat transfer coefficient to environment |
|  | °C | Environment temperature |
|  | °C | input temperature |

A simplified model of tank temperature dynamics can be written as

1. Write the system equations in state space form , where, the tank water temperature, and input the heater power.

**SOLUTION:**

In order for the model to make sense, all of its elements must have the same units. The left hand side has units of power (Watts), that is the energy variation of the system in Joules per second

:

All the other terms must therefore have the same units. If the specific heat has units then the flow must be converted to .

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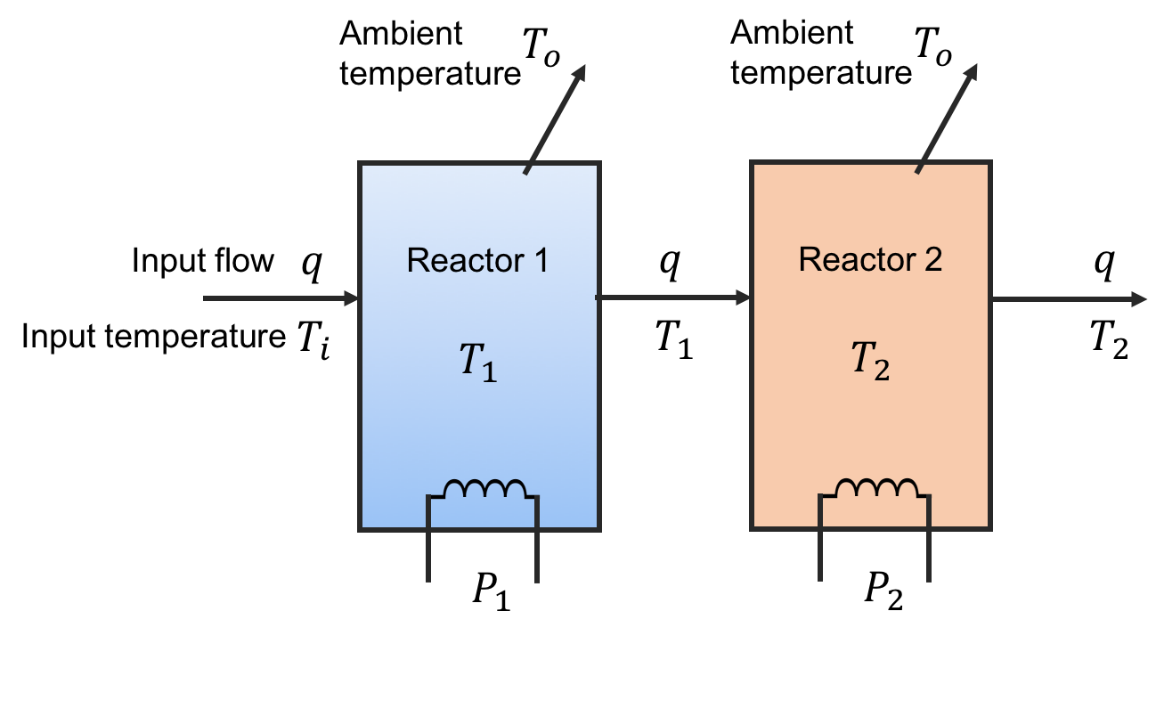
Now defining constants

The system can be written as

1. Write the system as a 1-dimensional differential equation in in state space form as , where **,** input vector **,**

**SOLUTION:**

1. Consider now the case in which there are two reactors connected in series, and the inlet of the second reactor is connected to the output of the first one. Assume also that both reactors are identic, and are located in the same ambient temperature. Write the system equations as a 2-dimensional differential equation in in state space form as , where is the state vector containing the temperatures of reactors 1 and 2, **,** the input vector **,** contains the input temperature to the first reactor, the ambient temperature, and the applied heater powers of reactors and . The input matrix **.**

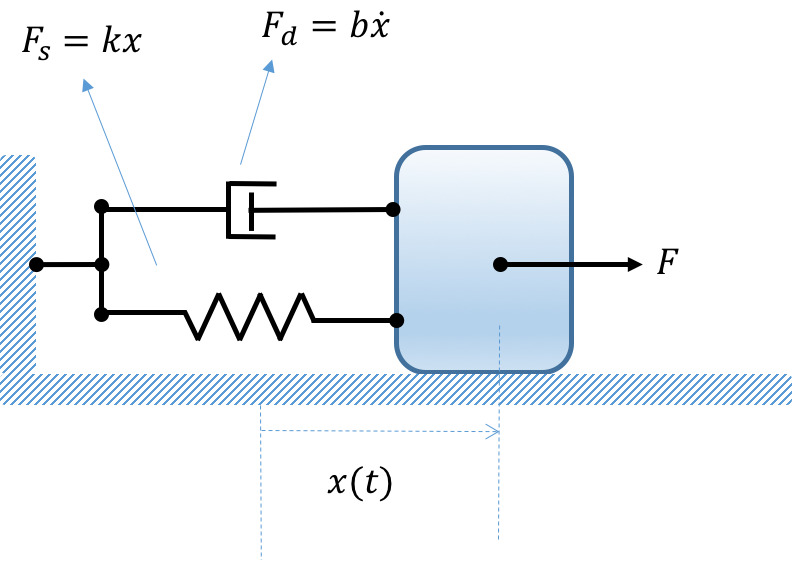


SOLUTION:

1. Simulate the system using forward Euler method. Use initial tank temperature °C, 500W, 300W. Provide M-script used. Make a plot of tank temperature versus time. Label axis accordingly with appropiate units. Note that the flow is given in liters/second so you will have to convert it to SI units (m^3/s) for the equations to be correct.
2. Experiment with different values of and flow q. Make a plot which shows the the solution of the system with a least two sets of values. Hint: use hold on to overlay several time series in a single plot.
3. Simulate the system using ode45. Repeat step (4) but now using ode45. Provide m-scripts as well as a plot of temperature vs time.

# Mass spring damper system

Consider a simplified model of a mass spring damper system. A mass is connected with a spring and a damper to a fixed point and can slide in a friction-less surface. The spring force is proportional to the mass displacement with respect to the equilibrium (when =0, the spring force is zero). The damper generates a force with sign opposed to the mass velocity and proportional to the velocity .



1. Write the system equations in state space form , where, , . (Note the difference between bold symbol , used to represent the state vector,and position which does not use bold). (Note: Remember that we will try to use this notation during the course: bold symbols represent vector or matrix variables, whereas non bold symbols represent scalar variables).

SOLUTION:

1. What are the units of all the involved variables and parameters ,, , ? Verify that the simplified mathematical model is consistent with respect to the units.

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|  |  |

In order for the model to make sense, all of its elements must have the same units

:

:

:

:

1. Write the system equations in state form as a linear system , where **,**

SOLUTION:

Note: In the question it was not specified that . This is the most natural choice of input for the system. But you could also have chosen , and then .

1. Implement in Matlab a Forward-Euler integrator to simulate the system. Provide m-scripts used. Use the following values

|  |  |  |
| --- | --- | --- |
|  | 1m | Initial position |
|  | 0m/s | Initial velocity |
|  | 1kg | mass |
|  | 1 N/m | Spring constant |
|  | 1 Ns/m | Damper constant |
|  | 5 N | External force |

* 1. Plot position, velocity and acceleration as a function of time. Use
  2. Experiment with different time steps . Does it make any difference? What happens if you use a too large time step?

1. Repeat the simulation now with values:

|  |  |  |
| --- | --- | --- |
|  | 1m | Initial position |
|  | 0m/s | Initial velocity |
|  | 1kg | mass |
|  | 1 N/m | Spring constant |
|  | 4 Ns/m | Damper constant |
|  | 5 N | External force |

Do you see any difference in system behavior? Provide plots to justify your answer.

1. Implement the system equations using Matlab Simulink. Compare the results with the forward Euler method. Do the results agree? Provide screen captures of the Simulink model used, and plots using the smulink signal scope for position and velocity.

SOLUTION:

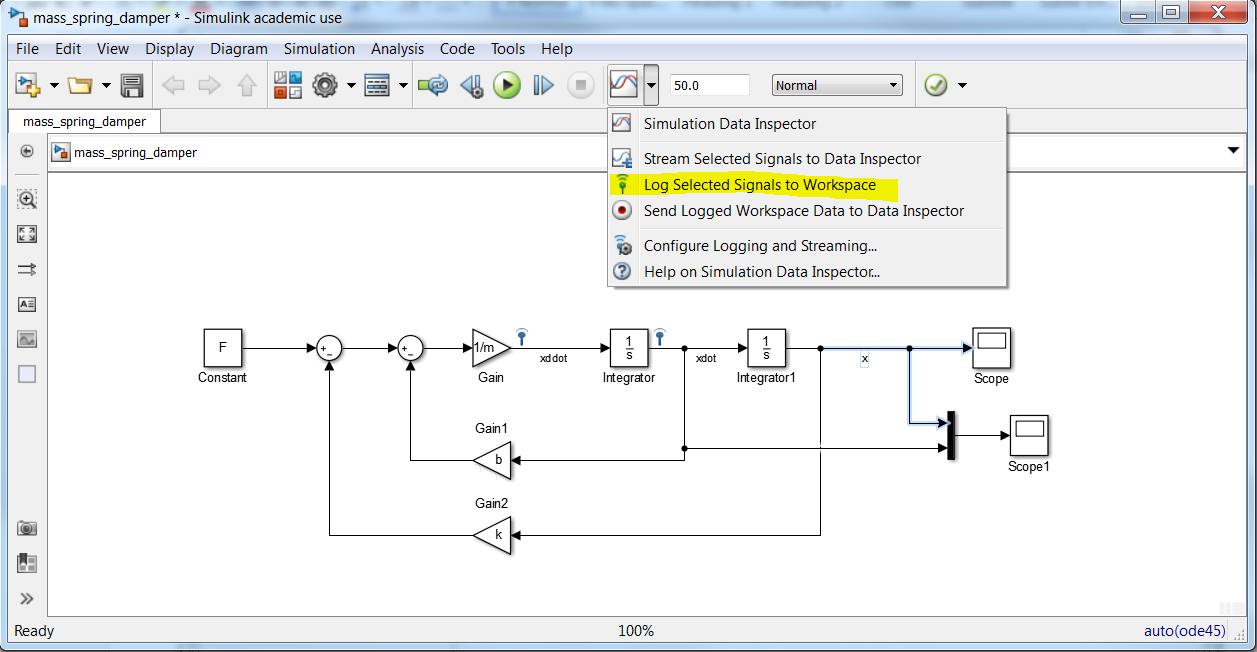


Figure 1: You can configure signals to log right-clicking on a signal and selecting Log selected Signals to Workspace

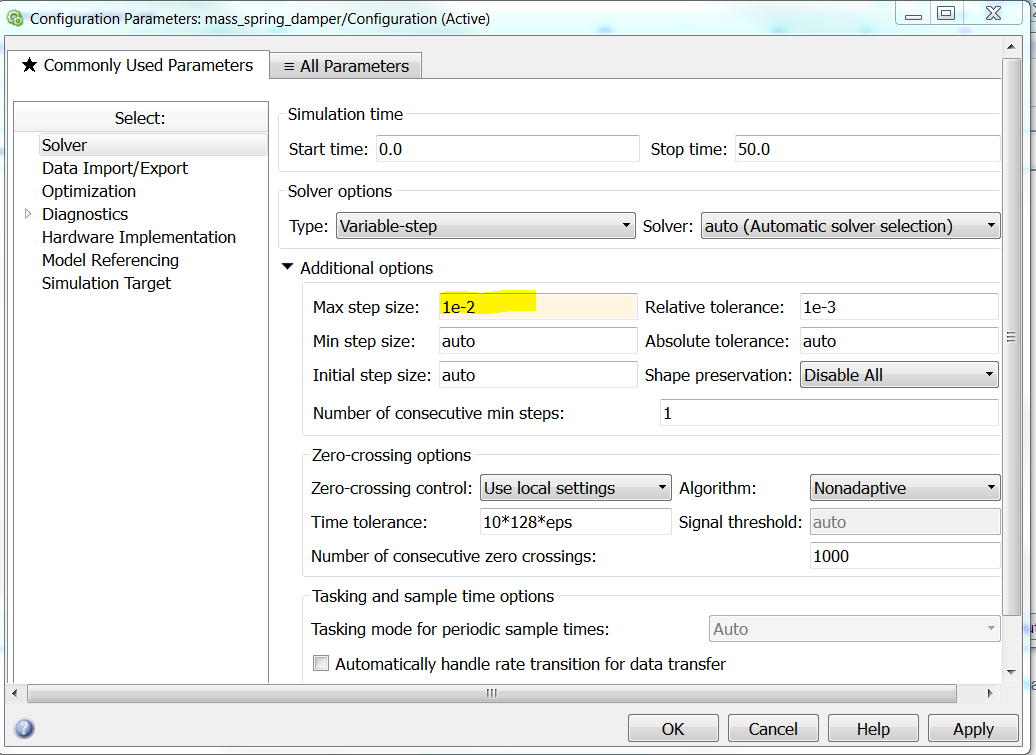


Figure 2: You can set the maximum step size in order to have more data points. Otherwise Simulink selects step size which computes the fastest solution within a given error.

1. Leaving the rest of parameters fixed, can you find the value of damper constant in which the system starts/stops oscillating?

SOLUTION:

The goal was that you tried different values of to see how this affects the solution. You should have found that when is approximately greater than 2, then the system does not oscillate. As gets smaller than that the system starts oscillating. And when the system behaves as a perfect oscillator.

Later in the course you will learn that the mass spring damper system is a second order system which has a transfer function from force to position given by

The poles of the system, the roots of the denominator are given by solving

If the poles are imaginary then there is oscillation in the output. If the poles are real then the system has no oscillation. The poles are real if

Which is true if

And using the exercise parameters

# Car traffic in one lane (optional exercise)

This is an optional exercise, not compulsory. In this exercise you will simulate a model of a group of cars driving on a single lane and trying to keep a pre-defined distance to the next car. Suppose that there are cars driving on a single lane. The position of the first car is denoted by , the second car and the last car by . The velocity of the first car is denoted by the second car and the velocity of the last car by .



The distance between the first and the second car is denoted by . When the drivers took their driving certificate, they were taught to keep a distance to the following car such as it takes 3 seconds to reach the following car if it instantaneously stops. That is, the ideal distance that the second car is trying to keep with respect to the first car is given by

Generally the driver in the ’th car is trying to keep a distance to the next car of

The actual distance between the ’th car and the next is given by . A driver uses the gas pedal to modify its car velocity such as to minimize the difference . This can be modelled by a first order differential equation (later in the course you will learn that this is a simple proportional-controller)

where is a constant that depends on the driver, and models how aggressive is his response in order to try to keep the desired distance to the next car.

1. Consider that you are asked to simulate the system and build a state space model with the following state

which is obtained by stacking all the car positions and velocities in a column vector. Also, consider the acceleration of the first car as the input, . Write the system equations in state space form

**SOLUTION:**

1. Write the system equations in the form of a continuous time linear system , where **,**

**SOLUTION:**

1. Simulate the system using forward Euler method. Chose car starting positions, the initial velocity of the first car. Assume the first car keeps constant velocity Choose driver constants
2. Repeat now using a sinusoidal acceleration for the first car
3. In the video “The simple solution to traffic” <https://www.youtube.com/watch?v=iHzzSao6ypE> it is suggested that by choosing a very different driving strategy, it could be possible to reduce traffic jams. The strategy is that each driver tries to keep an equal distance between the next and previous car. As a first approach assume that the drivers can be modelled by a simple proportional control law in which the driver tries to keep the difference equal to zero, where is the distance to the next car:

And is the distance to the car behind:

where again is a driver dependent constant which represents how aggressive is the driver in following the rule. The first vehicle follows where is considered an input signal. The last vehicle can not compute distance to the car behind, therefore you can assume that the last vehicle follows the standard 3 seconds distance rule:

Write the system equations in state space form with the acceleration of the first car.

**SOLUTION:**

1. Simulate the system using forward Euler method. Chose car starting positions, the initial velocity of the first car. Assume the first car keeps constant velocity Choose driver constants